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Robust H_∞ control for linear Markovian jump systems with unknown nonlinearities

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Abstract

This paper studies the problem of stochastic stability and disturbance attenuation for a class of linear continuous-time uncertain systems with Markovian jumping parameters. The uncertainties are assumed to be nonlinear and state, control and external disturbance dependent. A sufficient condition is provided to solve the above problem. An H_∞ controller is designed such that the resulting closed-loop system is stochastically stable and has a disturbance attenuation γ for all admissible uncertainties. It is shown that the control law is in terms of the solutions of a set of coupled Riccati inequalities. A numerical example is included to demonstrate the potential of the proposed technique.

Keywords: H_∞ control; Markovian jump parameter; Riccati inequality; Stochastic stability; Uncertainty

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1. Introduction

The important problem in control system synthesis is the design of a controller such that the closed-loop system is internally stochastically stable and the effect of the disturbance input signals on some desired output signals is attenuated. This problem has been extensively studied for linear systems, see, for example, [6,12] and references therein. For nonlinear systems we refer the reader to [1,9,14,19].

The class of systems with Markovian jump parameters represents an interesting class of systems that we can use to model a variety of physical systems. This class of systems has two components in the state vector. The first one which varies continuously is referred to be the continuous state of the system and the second one which varies discretely is referred to be the mode of the system. This kind of systems has been widely used, for instance, manufacturing systems [20] and communication systems [15] and references therein.

Systems with Markovian jumping parameters has been initially introduced by Krasovskii and Lidskii [13] in which the framework of this class of systems is stated. Since then the research on this field has been dramatically developed for the last three decades. Among them, we quote [5,11,15] and the references therein. Linear systems with Markovian jumping parameters has been extensively studied. Now, a great number of results are available to be used to control physical systems. For some representative prior work on this general topic, we refer the reader to [11,22]. Recently, H_∞ control problem for linear continuous-time systems with Markovian jumping parameters has been addressed in the work of [7,17,22,26]. However, to the best of our knowledge, to date, the problem on the stochastic stability and stochastic stabilizability of nonlinear systems have not been drawn much attention yet.

In this paper, we deal with the robust stochastic stabilizability and disturbance attenuation for a class of linear continuous-time systems with Markovian jumping parameters. The system under consideration has a linear nominal part with an unknown nonlinearity. The first part of the uncertainties is assumed to be norm bounded and satisfy the matching condition. The second part is a function of a norm bounded external disturbance. The third component of the uncertainty is state and mode dependent and normed bounded. Via Riccati equation approach, a controller is designed such that the closed-loop uncertain system is stochastically stable and has a disturbance attenuation γ for all admissible uncertainties and unknown nonlinearities. It is shown that the above problem can be solved if a set of coupled Riccati-like inequalities has symmetric positive definite solutions.

The paper is organized as follows. In Section 2, the problem is formulated and some preliminary results are recalled. In Section 3, a sufficient condition is proposed under which the closed-loop uncertain system is robustly stable with H_∞-norm bound γ. In Section 4, a numerical example is given to show the usefulness of the proposed results.

Notation. The notations in this paper are fairly standard. $\Re^n$ and $\Re^{n\times m}$ denote, respectively, the n-dimensional Euclidean space and the set of all $n \times m$ real matrices.

The superscript "T" denotes the transpose and the notation $X \geqslant Y$ (respectively, $X > Y$) where X and Y are symmetric matrices, means that $X - Y$ is positive semidefinite (respectively, positive definite). I is the identity matrix with appropriate dimension. $\mathrm{E}\{\cdot\}$ denotes the expectation operator with respective to some probability measure P. $L^2[0,T]$ stands for the space of square integrable vector functions over the interval $[0,T]$. $\|\cdot\|$ will refer to either the Euclidean vector norm or the matrix norm which is the operator norm induced by the standard vector norm. $\|\cdot\|_2$ stands for the norm in $L^2[0,T]$, while $\mathrm{E}\|\cdot\|$ denotes the norm in $L^2((\Omega,\mathcal{F},P),[0,T])$. $(\Omega,\mathcal{F},P)$ is a probability space. $\lambda_{\min}(A)$ stands for the minimal eigenvalue of the matrix A.

2. Problem statement

Let us assume that the class of systems we consider in this paper be described by the following nonlinear differential equations:

$$\dot{x}(t) = A\big(r(t)\big)x(t) + B\big(r(t)\big)u(t) + F\big(x(t),u(t),r(t),w(t),t\big),$$
$$x(0) = x_0,\ r(0) = r_0, \tag{2.1}$$
$$z(t) = C\big(r(t)\big)x(t), \tag{2.2}$$

where $x(t) \in \mathbb{R}^n$ is the state vector of the system at time t, $u(t) \in \mathbb{R}^m$ is the control input of the system, $w(t) \in \mathbb{R}^m$ is the bounded disturbance belonging to $\mathcal{L}_2[0,\infty)$, $z(t) \in \mathbb{R}^p$ is the penalty variable related to some performance cost, and $r(t)$ is a continuous-time Markov process taking values in a finite state space denoted by $\mathcal{S} = \{1,2,\ldots,s\}$; $F(x(t),r(t),w(t),t) \in \mathbb{R}^n$ is the system uncertainty; $A(r(t))$, $B(r(t))$, and $C(r(t))$ are given constant matrices for each value of $r(t)$ in $\mathcal{S}$; $x(0) = x_0$ and $r(0) = r_0$ are, respectively, the initial values of the state and the mode at time $t = 0$.

The evolution of the stochastic process $\{r(t),\ t \geqslant 0\}$ that determines the mode of the system at each time t is assumed to be described by the following probability transitions:

$$P\big[r(t+h) = \beta \mid r(t) = \alpha\big] = \begin{cases} q_{\alpha\beta}h + o(h) & \text{if } \alpha \neq \beta, \\ 1 + q_{\alpha\sigma}h + o(h) & \text{otherwise,} \end{cases} \tag{2.3}$$

with $q_{\alpha\beta} \geqslant 0$ for all $\alpha \neq \beta$ and $q_{\alpha\alpha} = -\sum_{\beta\in\mathcal{S},\ \beta\neq\alpha} q_{\alpha\beta}$ for all $\alpha \in \mathcal{S}$, and $\lim_{h\to 0}\frac{o(h)}{h} = 0$.

Remark 2.1. System (2.1)–(2.3) can be used to model many physical systems in which the nominal part describes the linearization of the real physical system and the uncertain term $F(x(t),u(t),r(t),w(t),t)$ represents the different errors and uncertainties, like the linearization errors, the external disturbance, etc. The presence of the stochastic parameter $r(t)$ in the model can be justified by the fact that the real system can have many operating points that depend on the state of the system.

Remark 2.2. Note that system (2.1)–(2.3) is a hybrid system in which one state $x(t)$ takes values continuously and another "state" $r(t)$ takes values discretely. This kind of system can be used to represent many important physical systems subject to random failures and structure changes, such as electric power systems, control systems of a solar thermal central receiver, communications systems, aircraft flight control, and manufacturing systems; see, for example, [2,3] and references therein.

The problem we address in this paper is how we can design a control law that stabilizes the system, attenuates the effect of the external disturbance, and guarantees its robustness. We are also interested in the conditions under which this will be true.

Let $x(t, x_0, r_0)$ denote the trajectory of the state $x(t)$ from the initial state (x_0, r_0). We introduce the following stochastic stability and stochastic stabilizability concepts for continuous-time nonlinear systems with Markovian jumping parameters.

Definition 2.1. For system (2.1) with $u(t) \equiv 0$ and $F(x(t), r(t), w(t), t) \equiv 0$, for all $r(t) \in \mathcal{S}$, the equilibrium point 0 is *stochastically stable*, if for every initial state (x_0, r_0),

$$\mathrm{E}\left\{\int_0^\infty \left\|x(t, x_0, r_0)\right\|^2\right\} dt < \infty \tag{2.4}$$

holds.

Definition 2.2. We say that system (2.1) is *stochastically stabilizable*, if for every initial state (x_0, r_0), there exists a feedback control law $u(t) = -k(x(t), r(t))$, such that the closed-loop system

$$\dot{x}(t) = A\big(r(t)\big)x(t) - B\big(r(t)\big)k\big(r(t), x(t)\big) + F\Big(x(t), -k\big(x(t), r(t)\big), r(t), w(t), t\Big)$$

is stochastically stable for all admissible uncertainty (it will be stated in Assumptions 2.1, 2.2 and 2.4) $F(x(t), -k(x(t), r(t)), r(t), w(t), t)$.

Definition 2.3. Consider system (2.1)–(2.3). Given $\gamma > 0$, the mapping from $w(t)$ to $z(t)$ is said to have $\mathcal{L}_2$-gain less than or equal to γ if for all initial state x_0, $x_0 < \infty$, mode r_0, and fixed control law $u(t)$,

$$\mathrm{E}\left[\int_0^T \left\|z(t)\right\|^2 dt\right] \leqslant \gamma^2 \int_0^T \left\|w(t)\right\|^2 dt \tag{2.5}$$

holds for all $T \geqslant 0$ and for all admissible uncertainties.

Associated with system (2.1)–(2.3), if inequality (2.5) holds, this systems is also said to be *dissipative* (with respect to the supply rate $\mathrm{E}\,\|z(t)\|^2 - \gamma^2\|w(t)\|^2$), for the concept of dissipative of deterministic system, much progress has been made since the work of Willems [27].

Definition 2.4. Given $\gamma > 0$, system (2.1)–(2.3) is said to be stochastically stable with disturbance attenuation γ for $T \to \infty$, if it is stochastically stable and dissipative, i.e., inequality (2.5) holds.

Definition 2.5. Given $\gamma > 0$, system (2.1) is said to be dissipative with respect to the supply rate $\|z(t)\|^2 - \gamma^2\|w(t)\|^2$ if for all $T > 0$, we have

$$\mathrm{E}\left[\int_0^T \left(\left\|z(s)\right\|^2 - \gamma^2\left\|w(s)\right\|^2\right) ds\right] \leqslant 0. \tag{2.6}$$

Definition 2.6. Given $\gamma > 0$, a function $v(x(t), r(t))$ is called a storage function of system (2.1)–(2.3) if it satisfies $v(0, r_0) = 0$, $v(x(t), r(t)) \geqslant 0$ for all r_0, $x(t)$, and $r(t)$, and

$$\mathrm{E}\left[\int_{t_0}^{t_1} \|z(s)\|^2\, ds\right] - \gamma^2 \int_{t_0}^{t_1} \|w(s)\|^2\, ds \leqslant v\big(x(t_0), r(t_0)\big) - v\big(x(t_1), r(t_1)\big) \tag{2.7}$$

along any trajectory of system (2.1)–(2.3) for all $t_1 \geqslant t_0$ and all $w(\cdot) \in \mathcal{L}_2[t_0, t_1]$.

In connection to system (2.1)–(2.3), we have the following lemma which establishes the links between the concepts of dissipativity and storage function.

Lemma 2.1. *System* (2.1)–(2.3) *is dissipative if there exists a storage function for this system.*

Proof. It can be worked out via a similar technique as that used in [8] for deterministic version without jump parameters. The details are omitted. □

In the following assumptions, we will give the structure of the uncertainties and the required hypothesis for the rest of the paper.

Assumption 2.1. Let us assume that the system uncertainty $F(x(t), u(t), r(t), w(t), t)$ is given by

$$\begin{aligned} F\big(x(t), u(t), r(t), w(t), t\big) &= \Delta A\big(x(t), r(t), t\big)x(t) + \Delta B\big(x(t), r(t), t\big)u(t) \\ &\quad + B_1\big(r(t)\big)w(t) + B\big(r(t)\big) f\big(x(t), r(t), t\big), \end{aligned} \tag{2.8}$$

where $\Delta A(x(t), r(t), t)$ and $\Delta B(x(t), r(t), t)$ are matrix functions representing the system uncertainties in the matrices $A(r(t))$ and $B(r(t))$, respectively, $B_1(r(t))$ is a constant known matrix for each value of $r(t)$ in $\mathcal{S}$ and $f(x(t), r(t), t)$ is an $m \times 1$ vector representing the nonlinear uncertainties in the autonomous part of the system.

The following assumption is introduced for uncertainties $\Delta A(x(t), r(t), t)$ and $\Delta B(x(t), r(t), t)$.

Assumption 2.2. Let $D(r(t))$ and $E(r(t))$ be two known real constant matrices for each value of $r(t)$ in $\mathcal{S}$. Let us assume that the uncertainties $\Delta A(x(t), r(t), t)$ and $\Delta B(x(t), r(t), t)$ have the following forms:

$$\Delta A\big(x(t), r(t), t\big) = D\big(r(t)\big) G\big(x(t), r(t), t\big) E\big(r(t)\big), \tag{2.9}$$

$$\Delta B\big(x(t), r(t), t\big) = B\big(r(t)\big) J\big(x(t), r(t), t\big), \tag{2.10}$$

where $G(x(t), r(t), t)$ and $J(x(t), r(t), t)$ are Carathéodory matrix[3] functions bounded by

[3] A function $V: \mathbb{R}^n \times \mathcal{S} \times \mathbb{R}$ is called Carathéodory if (i) $V(z, r(t), \cdot)$ is Lebesgue measurable for each $z \in \mathbb{R}^n$ and for any $r(t) = i \in \mathcal{S}$; (ii) $V(\cdot, r(t), t)$ is continuous for each $t \in \mathbb{R}$ and for any $r(t) = i \in \mathcal{S}$; (iii) for each compact set $U \subset \mathbb{R}^n \times \mathcal{S} \times \mathbb{R}$, there exists a Lebesgue integrable function $m_u(t)$ such that $\|V(z, r(t), t)\| \leqslant m_u(t)$ for all $(z, r(t), t) \in U$.

$$G^{\mathrm{T}}\big(x(t), r(t), t\big)G\big(x(t), r(t), t\big) \leqslant \eta_0 I, \tag{2.11}$$

$$\max_{(x(t),r(t),t)\in\mathbb{R}^n\times\mathcal{S}\times\mathbb{R}} \big\| J\big(x(t), r(t), t\big)\big\| \leqslant \eta_1, \tag{2.12}$$

where $\eta_0 \geqslant 0$ and $0 \leqslant \eta_1 < 1$ are two known constant scalars.

Remark 2.3. The parameter uncertainty structure as in (2.9) is an extension of the so-called "matching condition" of (2.10), which has been widely used in the problems of robust control and robust filtering of uncertain systems (see, e.g., [18,21–25,28] and references therein) and many practical systems possess parameter uncertainties which can be either exactly modeled, or overbounded by (2.10). The matrices $D(r(t))$ and $E(r(t))$ specify how the uncertain parameters in $G(x(t), r(t), t)$ affects the nominal matrices of system (2.1)–(2.2).

Assumption 2.3. Let us assume that the nominal system (with $\Delta A(x(t), r(t), t)$, $\Delta B(x(t), r(t), t)$, and $f(x(t), r(t), t)$ all set to zero) is stochastically stabilizable.

Assumption 2.4. There exists a positive Carathéodory function $\rho(x(t), r(t), t)$ such that $\|f(x(t), r(t), t)\| \leqslant \rho(x(t), r(t), t)$ for all $(x(t), r(t), t) \in \mathbb{R}^n \times \mathcal{S} \times \mathbb{R}$, where $\|\cdot\|$ denotes the Euclidean norm. Also, $\rho(0, r(t), t) = 0$ and $\lim_{t\to\infty} \rho(x(t), r(t), t) < \infty$ for all $(x(t), r(t)) \in \mathbb{R}^n \times \mathcal{S}$.

Before ending this section, let us recall the following inequality which will be used in the proof of our main result.

Lemma 2.2 [18]. *Given matrices H, F, and E of appropriate dimensions with $FF^{\mathrm{T}} \leqslant \alpha I$, where $\alpha > 0$. Then for any $\varepsilon > 0$, we have*

$$HFE + E^{\mathrm{T}}F^{\mathrm{T}}H^{\mathrm{T}} \leqslant \varepsilon\alpha HH^{\mathrm{T}} + \frac{1}{\varepsilon}E^{\mathrm{T}}E.$$

3. Robust stabilization

Let us now return to the optimization problem we formulated in Section 2. Our goal in this section is to design a control law for the corresponding uncertain system (2.1)–(2.3) under the previous assumptions that robustly stabilizes the system and rejects the effect of the disturbance $w(t)$.

Theorem 3.1. *Consider system* (2.1)–(2.3) *and given a scalar $\gamma > 0$. If there exist a set of $\varepsilon(i) > 0$, $i = 1, 2, \ldots, s$, and a set of symmetric and positive-definite matrices $Q(i)$, $i = 1, 2, \ldots, s$, such that the algebraic Riccati inequality*

$$\begin{aligned}
&A^{\mathrm{T}}\big(r(t)\big)P\big(r(t)\big) + P\big(r(t)\big)A\big(r(t)\big) + \sum_{\beta\in\mathcal{S}} q_{r(t)\beta}P(\beta)\\
&\quad + P\big(r(t)\big)\big[\varepsilon\big(r(t)\big)\eta_0 D\big(r(t)\big)D^{\mathrm{T}}\big(r(t)\big)\\
&\qquad\qquad - 2B\big(r(t)\big)B^{\mathrm{T}}\big(r(t)\big) + \gamma^{-2}B_1\big(r(t)\big)B_1^{\mathrm{T}}\big(r(t)\big)\big]P\big(r(t)\big)\\
&\quad + \frac{1}{\varepsilon(r(t))}E^{\mathrm{T}}\big(r(t)\big)E\big(r(t)\big) + C^{\mathrm{T}}\big(r(t)\big)C\big(r(t)\big) + Q\big(r(t)\big) \leqslant 0
\end{aligned} \tag{3.1}$$

has a set of symmetric positive-definite solutions $P(i)$, $i = 1, 2, \dots, s$. Then, there exists a controller $u(t)$ such that

$$\mathcal{A}(v)\big(x(t), r(t), w(t)\big) = \mathcal{A}_v(v)\big(x(t), r(t)\big) + \big\|z(t)\big\|^2 - \gamma^2\big\|w(t)\big\|^2 \leqslant 0, \tag{3.2}$$

where $\mathcal{A}_v(v)(x(t), r(t))$ is the infinitesimal operator of the function $v(x(t), r(t), t) = x^{\mathrm{T}}(t)P(r(t))x(t)$ and its expression is given by

$$\begin{aligned}\mathcal{A}_v(v)\big(x(t), r(t)\big) = {} & x^{\mathrm{T}}(t)\big[\big[A\big(r(t)\big) + \Delta A\big(x(t), r(t), t\big)\big]^{\mathrm{T}} P\big(r(t)\big)\big]x(t) \\ & + x^{\mathrm{T}}(t)\big[P\big(r(t)\big)\big[A\big(r(t)\big) + \Delta A\big(x(t), r(t), t\big)\big]\big]x(t) \\ & + 2x^{\mathrm{T}}(t)P\big(r(t)\big)\big[B\big(r(t)\big) + \Delta B\big(x(t), r(t), t\big)\big]u(t) \\ & + 2x^{\mathrm{T}}(t)P\big(r(t)\big)B_1\big(r(t)\big)w(t) \\ & + 2x^{\mathrm{T}}(t)P\big(r(t)\big)B\big(r(t)\big)f\big(x(t), r(t), t\big) \\ & + \sum_{\beta\in\mathcal{S}} q_{r(t)\beta}P(\beta).\end{aligned} \tag{3.3}$$

Moreover, a suitable controller can be chosen as

$$u(t) = -K\big(r(t)\big)x(t) - \frac{1}{1-\eta_1}\Phi\big(x(t), r(t), t\big), \tag{3.4}$$

where $\Phi(x(t), r(t), t)$ and $K(r(t))$ are given by

$$\Phi\big(x(t), r(t), t\big) = \frac{K(r(t))x(t)[\rho(x, r(t), t) + \eta_1\|K(r(t))x(t)\|]^2}{\|K(r(t))x(t)\|[\rho(x, r(t), t) + \eta_1\|K(r(t))x(t)\|] + \varepsilon^\star\|x\|^2}, \tag{3.5}$$

$$K\big(r(t)\big) = B^{\mathrm{T}}\big(r(t)\big)P\big(r(t)\big), \tag{3.6}$$

where η_1 is given by Eq. (2.12) *and $\varepsilon^\star$ is a positive scalar satisfying*

$$0 < \varepsilon^\star < \frac{\lambda_{\min}[Q(r(t))]}{2}. \tag{3.7}$$

Proof. The proof essentially follows a similar line to the proof of a result in the work of Nguang [16] for nonlinear systems without jump parameters. Let us assume that the Riccati equation (3.1) has a solution $P = (P(1), \dots, P(s))$ which is symmetric and positive-definite for some given $\varepsilon = (\varepsilon(1), \dots, \varepsilon(s)) > 0$ and $Q = (Q(1), \dots, Q(s))$ symmetric and positive-definite. Let $v(x(t), r(t)) = x^{\mathrm{T}}(t)P(r(t))x(t)$ be a candidate Lyapunov function for system (2.1)–(2.3) with the control law given by Eq. (3.4).

We need to show that

$$\mathcal{A}(v)\big(x(t), r(t), w(t)\big) = \mathcal{A}_v(v)\big(x(t), r(t)\big) + \big\|z(t)\big\|^2 - \gamma^2\big\|w(t)\big\|^2 \leqslant 0, \tag{3.8}$$

where $\mathcal{A}_v(v)(x(t), r(t), w(t))$ is given by (3.3).

By substituting the control law $u(t)$ given in Eq. (3.4), one has the following expression for $\mathcal{A}_v(x(t), r(t), w(t))$:

$$\begin{aligned}\mathcal{A}_v(v)\big(x(t), r(t)\big) = {} & f_1\big(x(t), r(t), t\big) + f_2\big(x(t), r(t), t\big) + f_3\big(x(t), r(t), t\big) \\ & - z^{\mathrm{T}}(t)z(t) + \gamma^2 w^{\mathrm{T}}(t)w(t),\end{aligned} \tag{3.9}$$

where $f_1(x(t), r(t), t)$, $f_2(x(t), r(t), t)$, and $f_3(x(t), r(t), t)$ are defined as follows:

$$\begin{aligned}
f_1\big(x(t), r(t), t\big) &= x^{\mathrm{T}}(t)\big[\big[A\big(r(t)\big) + \Delta A\big(x(t), r(t), t\big)\big]^{\mathrm{T}} P\big(r(t)\big)\big]x(t) \\
&\quad + x^{\mathrm{T}}\big[P\big(r(t)\big)\big[A\big(r(t)\big) + \Delta A\big(x(t), r(t), t\big)\big]\big]x(t) \\
&\quad + \gamma^2 x^{\mathrm{T}}(t) P\big(r(t)\big) B_1\big(r(t)\big) B_1^{\mathrm{T}}\big(r(t)\big) P\big(r(t)\big) x(t) \\
&\quad + x^{\mathrm{T}}(t) C^{\mathrm{T}}\big(r(t)\big) C\big(r(t)\big) x(t) \\
&\quad - 2x^{\mathrm{T}}(t) P\big(r(t)\big) B\big(r(t)\big) K\big(r(t)\big) x(t) + \sum_{\beta \in \mathcal{S}} q_{r(t)\beta} P(\beta), \\
f_2\big(x(t), r(t), t\big) &= -2x^{\mathrm{T}}(t) P\big(r(t)\big)\bigg[\big[B\big(r(t)\big) + \Delta B\big(x(t), r(t), t\big)\big] \\
&\qquad \times \frac{1}{1-\eta_1} \Phi\big(x(t), r(t), t\big)\bigg] \\
&\quad - 2x^{\mathrm{T}}(t) P\big(r(t)\big)\big[\Delta B\big(x(t), r(t), t\big) K\big(r(t)\big) x(t) \\
&\qquad - B\big(r(t)\big) f\big(x(t), r(t), t\big)\big], \\
f_3\big(x(t), r(t), t\big) &= -\gamma^2\big[w(t) - \gamma^{-2} B_1^{\mathrm{T}}\big(r(t)\big) P\big(r(t)\big) x(t)\big]^{\mathrm{T}} \\
&\quad \times \big[w(t) - \gamma^{-2} B_1^{\mathrm{T}}\big(r(t)\big) P\big(r(t)\big) x(t)\big].
\end{aligned}$$

Note that the term $f_3(x(t), r(t), t)$ is always nonpositive for any $x(t)$ and $r(t)$. To verify our result in this theorem, it suffices to show that the terms $f_1(x(t), r(t), t)$ and $f_2(x(t), r(t), t)$ are also nonpositive.

For the term $f_1(x(t), r(t), t)$ by Lemma 2.2, one obtains

$$\begin{aligned}
&x^{\mathrm{T}}(t) P\big(r(t)\big) \Delta A\big(x(t), r(t), t\big) x(t) + x^{\mathrm{T}} \Delta A^{\mathrm{T}}\big(x(t), r(t), t\big) P\big(r(t)\big) x(t) \\
&\quad = x^{\mathrm{T}}(t) P\big(r(t)\big) D\big(r(t)\big) G\big(x(t), r(t), t\big) E\big(r(t)\big) x(t) \\
&\qquad + x^{\mathrm{T}} E^{\mathrm{T}}\big(r(t)\big) G^{\mathrm{T}}\big(x(t), r(t), t\big) D^{\mathrm{T}}\big(r(t)\big) P\big(r(t)\big) x(t) \\
&\quad \leqslant x^{\mathrm{T}}(t)\bigg[\varepsilon\big(r(t)\big)\eta_0 P\big(r(t)\big) D\big(r(t)\big) D^{\mathrm{T}}\big(r(t)\big) P\big(r(t)\big) \\
&\qquad\qquad + \frac{1}{\varepsilon(r(t))} E^{\mathrm{T}}\big(r(t)\big) E\big(r(t)\big)\bigg] x(t).
\end{aligned} \tag{3.10}$$

Substituting (3.1) into $f_1(x(t), r(t), t)$, one has

$$\begin{aligned}
f_1\big(x(t), r(t), t\big) &= x^{\mathrm{T}}(t)\big[\big[A\big(r(t)\big) + \Delta A\big(x(t), r(t), t\big)\big]^{\mathrm{T}} P\big(r(t)\big)\big]x(t) \\
&\quad + x^{\mathrm{T}}(t)\big[P\big(r(t)\big)\big[A\big(r(t)\big) + \Delta A\big(x(t), r(t), t\big)\big]\big]x(t) \\
&\quad + \gamma^2 x^{\mathrm{T}}(t) P\big(r(t)\big) B_1\big(r(t)\big) B_1^{\mathrm{T}}\big(r(t)\big) P\big(r(t)\big) x(t) \\
&\quad + x^{\mathrm{T}}(t) C^{\mathrm{T}}\big(r(t)\big) C\big(r(t)\big) x(t) \\
&\quad - 2x^{\mathrm{T}}(t) P\big(r(t)\big) B\big(r(t)\big) K\big(r(t)\big) x(t) + \sum_{\beta \in \mathcal{S}} q_{r(t)\beta} P(\beta) \\
&\leqslant x^{\mathrm{T}}(t)\big[A^{\mathrm{T}}\big(r(t)\big) P\big(r(t)\big) + P\big(r(t)\big) A\big(r(t)\big)\big] x(t)
\end{aligned}$$

$$
\begin{aligned}
&+ x^{\mathrm{T}}(t)\big[\varepsilon\big(r(t)\big)\eta_0 P\big(r(t)\big)D\big(r(t)\big)D^{\mathrm{T}}\big(r(t)\big)P\big(r(t)\big)\big]x(t) \\
&- 2x^{\mathrm{T}}(t)\big[P\big(r(t)\big)B\big(r(t)\big)B^{\mathrm{T}}\big(r(t)\big)P\big(r(t)\big)\big]x(t) \\
&+ x^{\mathrm{T}}(t)\bigg[\frac{1}{\varepsilon(r(t))}E^{\mathrm{T}}\big(r(t)\big)E_1\big(r(t)\big) \\
&\qquad + \gamma^{-2}P\big(r(t)\big)B_1\big(r(t)\big)B_1^{\mathrm{T}}\big(r(t)\big)P\big(r(t)\big)\bigg]x(t) \\
&+ x^{\mathrm{T}}(t)\bigg[C^{\mathrm{T}}\big(r(t)\big)C\big(r(t)\big) + \sum_{\beta\in\mathcal{S}} q_{r(t)\beta}P(\beta)\bigg]x(t).
\end{aligned} \tag{3.11}
$$

Bearing in mind inequality (3.1), it follows that

$$
f_1\big(x(t), r(t), t\big) \leqslant -x^{\mathrm{T}}(t)Q\big(r(t)\big)x(t). \tag{3.12}
$$

For the term $f_2(x(t), r(t), t)$, by using Assumption 2.2, we have

$$
\begin{aligned}
&f_2\big(x(t), r(t), t\big) \\
&\quad = -2x^{\mathrm{T}}(t)P\big(r(t)\big)\bigg[\big[B\big(r(t)\big) + \Delta B\big(x(t), r(t), t\big)\big]\frac{1}{1-\eta_1}\Phi\big(x(t), r(t), t\big)\bigg] \\
&\qquad - 2x^{\mathrm{T}}(t)P\big(r(t)\big)\big[\Delta B\big(x(t), r(t), t\big)K\big(r(t)\big)x(t) - B\big(r(t)\big)f\big(x(t), r(t), t\big)\big] \\
&\quad = -2x^{\mathrm{T}}(t)P\big(r(t)\big)B\big(r(t)\big)\big[I + J\big(x(t), r(t), t\big)\big]\frac{1}{1-\eta_1}\Phi\big(x(t), r(t), t\big) \\
&\qquad - 2x^{\mathrm{T}}(t)P\big(r(t)\big)B\big(r(t)\big)\big[J\big(x(t), r(t), t\big)K\big(r(t)\big)x(t) - f\big(x(t), r(t), t\big)\big] \\
&\quad \leqslant -2x^{\mathrm{T}}(t)P\big(r(t)\big)B\big(r(t)\big)\Phi\big(x(t), r(t), t\big) \\
&\qquad - 2x^{\mathrm{T}}(t)P\big(r(t)\big)B\big(r(t)\big)\big[J\big(x(t), r(t), t\big)K\big(r(t)\big)x(t) - f\big(x(t), r(t), t\big)\big].
\end{aligned} \tag{3.13}
$$

By substituting the function $\Phi(x(t), r(t), t)$ from (3.4) into (3.13), together with the technique employed in [16], one has

$$
\begin{aligned}
&f_2\big(x(t), r(t), t\big) \\
&\quad \leqslant 2x^{\mathrm{T}}(t)P\big(r(t)\big)B\big(r(t)\big)\big[f\big(x(t), r(t), t\big) - J\big(x(t), r(t), t\big)K\big(r(t)\big)x(t)\big] \\
&\qquad - \frac{\|K(r(t))x(t)\|^2[\rho(x(t), r(t), t) + \eta_1\|K(r(t))x(t)\|]^2}{\|K(r(t))x(t)\|[\rho(x(t), r(t), t) + \eta_1\|K(r(t))x(t)\|] + \varepsilon^\star\|x(t)\|^2} \\
&\quad \leqslant 2\big\{\big\|K\big(r(t)\big)x(t)\big\|\big[\rho\big(x(t), r(t), t\big) + \eta_1\big\|K\big(r(t)\big)x(t)\big\|\big]\big\} \\
&\qquad - 2\bigg\{\frac{\|K(r(t))x(t)\|^2[\rho(x(t), r(t), t) + \eta_1\|K(r(t))x(t)\|]^2}{\|K(r(t))x(t)\|[\rho(x(t), r(t), t) + \eta_1\|K(r(t))x(t)\|] + \varepsilon^\star\|x(t)\|^2}\bigg\} \\
&\quad = 2\bigg\{\frac{\|K(r(t))x(t)\|^2[\rho(x(t), r(t), t) + \eta_1\|K(r(t))x(t)\|]^2}{\|K(r(t))x(t)\|[\rho(x(t), r(t), t) + \eta_1\|K(r(t))x(t)\|] + \varepsilon^\star\|x(t)\|^2}\bigg\} \\
&\quad \leqslant 2\varepsilon^\star\big\|x(t)\big\|^2.
\end{aligned} \tag{3.14}
$$

Using now the different bounds of (3.12) and (3.14) on the functions $f_1(x(t), r(t), t)$, $f_2(x(t), r(t), t)$, respectively, and taking into account of the nonpositiveness of $f_3(x(t),$

$r(t), t)$, one obtains from (3.9),

$$\begin{aligned}
&\mathcal{A}_v(v)\big(x(t), r(t)\big) \\
&\quad \leqslant -x^{\mathrm{T}}(t)Q\big(r(t)\big)x(t) + 2\varepsilon^{\star}x^{\mathrm{T}}(t)x(t) - z^{\mathrm{T}}(t)z(t) + \gamma^2 w^{\mathrm{T}}(t)w(t) \\
&\quad = -z^{\mathrm{T}}(t)z(t) + \gamma^2 w^{\mathrm{T}}(t)w(t) - x^{\mathrm{T}}(t)\big[Q\big(r(t)\big) - 2\varepsilon^{\star}I\big]x(t). \qquad (3.15)
\end{aligned}$$

Substituting (3.15) into $\mathcal{A}(v)(x(t), r(t), t)$ yields

$$\mathcal{A}(v)\big(x(t), r(t), w(t)\big) \leqslant -x^{\mathrm{T}}(t)\big[Q\big(r(t)\big) - 2\varepsilon^{\star}I\big]x(t). \qquad (3.16)$$

By the assumption of $\varepsilon^{\star}$ in (3.7), inequality (3.16) is nonpositive for all $x(t)$, which completes the proof. □

The following lemma gives a sufficient condition for the dissipativity of system (2.1)–(2.3).

Lemma 3.1. *System* (2.1)–(2.3) *is dissipative if there exists a nonnegative function* $v(x(t), r(t)) = x^{\mathrm{T}}(t)P(r(t))x(t)$, $r(t) \in \mathcal{S}$, *such that* $\mathcal{A}(v)(x(t), r(t), w(t)) \leqslant 0$ *for all* $x(t) \in \mathbb{R}^n$, $r(t) \in \mathcal{S}$ *and* $w(t) \in \mathcal{L}_2[0, T]$, *where* $\mathcal{A}(v)(x(t), r(t), w(t))$ *is as in* (3.2) *of Theorem* 3.1.

Proof. It can be proven along the same line as in [27], together with now the random jump parameter $r(t)$ being taken into account. □

By Theorem 3.1 and Lemma 3.1, we have the following result.

Theorem 3.2. *Consider system* (2.1)–(2.3) *and given a scalar* $\gamma > 0$. *If all the conditions in Theorem* 3.1 *are true, then system* (2.1)–(2.3) *is robustly dissipative, that is, the mapping from* $w(t)$ *to* $z(t)$ *has a* $\mathcal{L}_2$*-gain less than or equal to* γ *for all* $T > 0$, $w(\cdot) \in \mathcal{L}_2(0, T]$, *and for all admissible parameter uncertainties.*

Our last result in this paper reads as follows.

Theorem 3.3. *Consider system* (2.1)–(2.3) *and given a scalar* $\gamma > 0$. *The system is stochastically stabilizable with disturbance attenuation* γ, *if all the conditions in Theorem* 3.1 *are true. Moreover, a suitable state feedback control law is given by* (3.4) *in Theorem* 3.1.

Proof. Under the conditions of Theorem 3.1, the statement of the uncertain system (2.1)–(2.3) has a disturbance attenuation γ can be guaranteed by Theorem 3.2. The rest of the proof will be focused on showing the stochastically stability of system (2.1)–(2.3).

By the assumption of $\varepsilon^{\star}$ in (3.7) of Theorem 3.1, we may define

$$V(x, r, w) = \mathrm{E}\left[\int_0^T x^{\mathrm{T}}(t)x(t)\,dt\right] - \alpha\gamma^2 \int_0^T w^{\mathrm{T}}(t)w(t)\,dt, \qquad (3.17)$$

where

$$\alpha = \frac{1}{\lambda_{\min}[Q(r(t)) - 2\varepsilon^\star I]}.$$

From (3.15) and taking (3.3) into account, by integrating this inequality from 0 to T and taking the mathematical expectation on both sides, one has for all admissible uncertainties

$$\begin{aligned}
&\mathrm{E}\left[\int_0^T \mathcal{A}_v\big(x(t), r(t)\big)\,dt\right] \\
&\quad\leqslant \mathrm{E}\left\{\int_0^T \left[-z^{\mathrm{T}}(t)z(t) + \gamma^2 w^{\mathrm{T}}(t)w(t) - x^{\mathrm{T}}(t)\left[Q\big(r(t)\big) - 2\varepsilon^\star I\right]x(t)\right]dt\right\}. \qquad (3.18)
\end{aligned}$$

Note that (see, for example, [10])

$$\mathrm{E}\left[\int_0^T \mathcal{A}_v\big(x(t), r(t)\big)\,dt\right] = \mathrm{E}\left[x^{\mathrm{T}}(T)P\big(r(T)\big)x(T) - x^{\mathrm{T}}(0)P\big(r(0)\big)x(0)\right]. \qquad (3.19)$$

Substituting (3.19) into (3.18), one obtains

$$\begin{aligned}
&\mathrm{E}\left\{\int_0^T \left[-\gamma^2 w^{\mathrm{T}}(t)w(t) + z^{\mathrm{T}}(t)z(t)\right]dt\right\} \\
&\quad\leqslant \mathrm{E}\left\{\int_0^T -x^{\mathrm{T}}(t)\left[Q\big(r(t)\big) - 2\varepsilon^\star I\right]x(t)\,dt\right. \\
&\qquad\qquad \left. - x^{\mathrm{T}}(T)P\big(r(T)\big)x(T) + x^{\mathrm{T}}(0)P\big(r(0)\big)x(0)\right\}. \qquad (3.20)
\end{aligned}$$

By (3.20), we have from (3.17) that

$$\begin{aligned}
&V(x, r, w) \\
&\quad= \mathrm{E}\left[\int_0^T x^{\mathrm{T}}(t)x(t)\,dt\right] - \alpha\gamma^2 \int_0^T w^{\mathrm{T}}(t)w(t)\,dt \\
&\quad= \mathrm{E}\left\{\int_0^T \left[x^{\mathrm{T}}(t)x(t) + \alpha\left[-\gamma^2 w^{\mathrm{T}}(t)w(t) + z^{\mathrm{T}}(t)z(t)\right] - \alpha z^{\mathrm{T}}(t)z(t)\right]dt\right\} \\
&\quad\leqslant \mathrm{E}\left\{\int_0^T \left[x^{\mathrm{T}}(t)x(t) - \alpha\left[x^{\mathrm{T}}(t)\left[Q\big(r(t)\big) - 2\varepsilon^* I\right]x(t) + z^{\mathrm{T}}(t)z(t)\right]\right]dt\right\} \\
&\qquad - \alpha\,\mathrm{E}\left[x^{\mathrm{T}}(T)P\big(r(T)\big)x(T) - x^{\mathrm{T}}(0)P\big(r(0)\big)x(0)\right]
\end{aligned}$$

$$\leqslant \mathrm{E}\left\{ \int_0^T x^{\mathrm{T}}(t)\big(I - \alpha\big[Q\big(r(t)\big) - 2\varepsilon^* I\big]\big)x(t)\,dt + \alpha x^{\mathrm{T}}(0)P\big(r(0)\big)x(0) \right\}$$
$$< \alpha\,\mathrm{E}\big[x^{\mathrm{T}}(0)P\big(r(0)\big)x(0)\big], \tag{3.21}$$

which implies that

$$\mathrm{E}\left\{ \int_0^T x^{\mathrm{T}}(t)x(t)\,dt \right\} < \gamma^2 \int_0^T w^{\mathrm{T}}(t)w(t)\,dt + \alpha\,\mathrm{E}\big[x^{\mathrm{T}}(0)P\big(r(0)\big)x(0)\big]. \tag{3.22}$$

Taking limit on both sides of (3.22) as $T \to \infty$, one has

$$\mathrm{E}\lim_{T\to\infty}\left\{ \int_0^T x^{\mathrm{T}}(t)x(t)\,dt \right\} \leqslant \lim_{T\to\infty} \gamma^2 \int_0^T w^{\mathrm{T}}(t)w(t)\,dt$$
$$+ \alpha\,\mathrm{E}\big[x^{\mathrm{T}}(0)P\big(r(0)\big)x(0)\big] < \infty,$$

which implies that system (2.1) is stochastically stable for all admissible parameter uncertainties, and the proof ends. □

Remark 3.1. Theorem 3.3 presents a sufficient condition for the robust stochastic stability and dissipativity of uncertain system (2.1)–(2.3), which is in terms of a set of coupled Riccati equations. Note that this condition may be conservative due to the use of upper bounds of $G(x(t), r(t), t)$ and $J(x(t), r(t), t)$. But this conservativeness can be improved by a appropriate selection of $\varepsilon(r(t))$, $r(t) \in \mathcal{S}$, discussed in [4].

Remark 3.2. Note that controller (3.4) designed in Theorem 3.1 is mode-dependent that may not be suitable to use in some real environments. We may obtain a sufficient condition for uncertain system (2.1)–(2.3) to be stochastically stable with $\mathcal{L}_2$-gain less than or equal to γ via a mode-independent controller by requiring that (3.1) is satisfied with $P(i)$, $i = 1, 2, \ldots, s$, that all equal to the same matrix P. We note that in such case the control law is independent of the transition probability matrix (q_{ij}), so we expect this condition to be quite conservative.

4. A numerical example

To show the usefulness of our model, let us consider a production system consisting of one machine producing one item. Let the Markov process $r(t)$ has two modes, i.e., $\mathcal{S} = \{1, 2\}$, and let its dynamics be described by the following nonlinear differential equations:

$$\dot{x}(t) = A\big(r(t)\big)x(t) + B_1 w + B_2 u + B_2 f\big(x_1, x_2, r(t)\big), \qquad z(t) = Cx(t) \tag{4.1}$$

with the transition matrix

$$Q = \begin{bmatrix} -0.1 & 0.1 \\ 0.5 & -0.5 \end{bmatrix},$$

$$A(1) = \begin{bmatrix} 0 & 1 \\ -1 & -1 \end{bmatrix}, \qquad A(2) = \begin{bmatrix} 0 & 1 \\ -1 & 1 \end{bmatrix},$$

$$B_1 = [0 \quad 0.1]^{\mathrm{T}}, \qquad B_2 = [1 \quad 1]^{\mathrm{T}}, \qquad C = [0 \quad 5],$$

$$\bigl|f\bigl(x(t),1\bigr)\bigr| \leqslant x_2^2(t), \qquad \bigl|f\bigl(x(t),2\bigr)\bigr| \leqslant x_2^4(t). \tag{4.2}$$

Let $\gamma = 1$ and $\varepsilon^\star = 0.01$. Then two solutions $P(i) > 0$, $i = 1, 2$, to (3.1) are

$$P(1) = \begin{bmatrix} 5.2502 & -1.6133 \\ -1.6133 & 2.7456 \end{bmatrix}, \qquad P(2) = \begin{bmatrix} 38.4880 & -30.1085 \\ -30.1085 & 27.7918 \end{bmatrix}$$

with

$$Q(1) = \begin{bmatrix} 1.6502 & 2.8938 \\ 2.8938 & 7.4200 \end{bmatrix}, \qquad Q(2) = \begin{bmatrix} 21.5929 & -10.9766 \\ -10.9766 & 11.4652 \end{bmatrix}.$$

Hence, a suitable controller is

$$u(t) = -K\bigl(r(t)\bigr)x(t) - \Phi\bigl(x(t), r(t), t\bigr), \tag{4.3}$$

where $\Phi(x(t), r(t), t)$ and $K(r(t))$ are given by

$$\Phi\bigl(x(t), r(t), t\bigr) = \frac{K(r(t))x(t)\rho^2(x, r(t), t)}{\|K(r(t))x(t)\|\rho(x, r(t), t) + \varepsilon^\star \|x\|^2}, \tag{4.4}$$

$$K\bigl(r(t)\bigr) = B_2^{\mathrm{T}} P\bigl(r(t)\bigr) \tag{4.5}$$

with

$$\rho(x, 1, t) = x_2^2(t), \qquad \rho(x, 2, t) = x_2^4(t).$$

For the disturbance input given in Fig. 1, the ratio of the energy of z to the energy of ω of system (4.1) with (4.3) is given in Fig. 2.

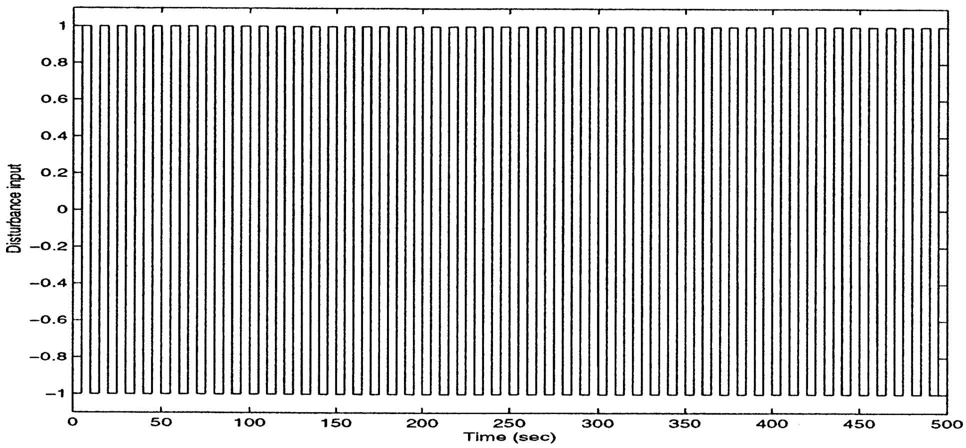

Fig. 1. The history of the disturbance input.

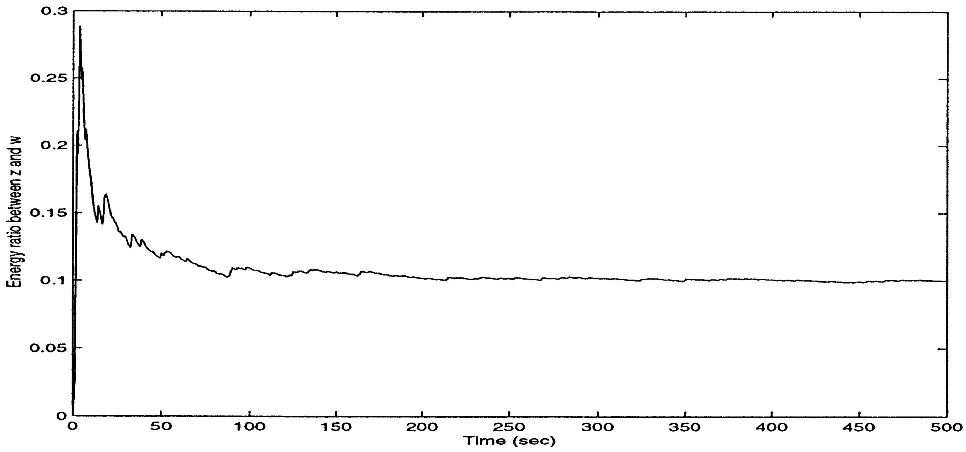

Fig. 2. The energy ratio between the penalty variable z and the disturbance input w.

Remark 4.1. From Fig. 2 we can see that after 200 seconds the ratio of the energy of z to the energy of ω of system (4.1) with (4.3) tends to a constant value which is about 0.1. So the $\mathcal{L}_2$ gain from ω to z is about $\sqrt{0.1} = 0.316$, which is less than the prescribed value 1.

5. Conclusion

In this paper, we investigated the problem of robust H_∞ control for a class of nonlinear continuous-time systems with both parametric uncertainties and Markovian jumping parameters. We designed a controller such that the uncertain system can be robustly stabilizable and a given disturbance attenuation can be achieved for all admissible uncertainties and unknown nonlinearities. We showed that this problem can be resolved via Riccati equation approach.

References

[1] J.A. Ball, J.W. Helton, M.L. Walker, H_∞ control for nonlinear systems with output feedback, IEEE Trans. Automat. Control 38 (1993) 546–559.
[2] E.K. Boukas, A. Haurie, Manufacturing flow control and preventive maintenance: a stochastic control approach, IEEE Trans. Automat. Control 35 (1990) 1024–1031.
[3] E.K. Boukas, Q. Zhang, G. Yin, Robust production and maintenance planning in stochastic manufacturing systems, IEEE Trans. Automat. Control 40 (1995) 1098–1102.
[4] E.K. Boukas, P. Shi, Stochastic stability and guaranteed cost control of discrete-time uncertain systems with Markovian jumping parameters, Internat. J. Robust Nonlinear Control 8 (1998) 1155–1167.
[5] O.L.V. Costa, M.D. Fragoso, Stability results for discrete-time linear systems with Markovian jumping parameters, J. Math. Anal. Appl. 179 (1993) 154–178.
[6] J.C. Doyle, K. Glover, P.P. Khargonekar, B.A. Francis, State space solutions to the standard H^2 and H^∞ control problems, IEEE Trans. Automat. Control 34 (1989) 831–847.
[7] V. Dragan, P. Shi, E.K. Boukas, Control of singularly perturbed systems with Markovian jump parameters: an H_∞ approach, Automatica 35 (1999) 1369–1378.

[8] D. Hill, P. Moylan, The stability of nonlinear dissipative systems, IEEE Trans. Automat. Control 21 (1976) 708–711.
[9] A. Isidori, A. Astolfi, Disturbance attenuation and H_∞ control via measurement feedback in nonlinear systems, IEEE Trans. Automat. Control 37 (1992) 1283–1293.
[10] K. Ito, H.P. McKean, Diffusion Processes and Their Sample Paths, Springer-Verlag, Berlin, 1965.
[11] Y. Ji, H.J. Chizeck, Controllability, stabilizability, and continuous-time Markovian jump linear quadratic control, IEEE Trans. Automat. Control 35 (1990) 777–788.
[12] P.P. Khargonekar, K.M. Nagpal, K.R. Poolla, H^∞ control with transients, SIAM J. Control Optim. 29 (1991) 1373–1393.
[13] N.N. Krasovskii, E.A. Lidskii, Analysis design of controller in systems with random attributes—Part 1, Automat. Remote Control 22 (1961) 1021–1025.
[14] W. Lin, C.I. Byrnes, Discrete-time nonlinear H_∞ control with measurement feedback, Automatica 31 (1995) 419–434.
[15] M. Mariton, Jump Linear Systems in Automatic Control, Dekker, New York, 1990.
[16] S.K. Nguang, Global robust H_∞ control of a class of nonlinear systems, Internat. J. Robust Nonlinear Control 7 (1997) 75–84.
[17] Z. Pan, T. Basar, Random evolutionary time-scale decomposition in robust control of jump linear systems, in: Proc. 35th IEEE Conf. on Decision and Control, Kobe, Japan, 1996, pp. 505–510.
[18] I.R. Petersen, A stabilization algorithm for a class of uncertain linear systems, Systems Control Lett. 8 (1987) 351–357.
[19] A.J. van der Schaft, L_2-gain analysis of nonlinear systems and nonlinear state feedback H_∞ control, IEEE Trans. Automat. Control 37 (1992) 770–784.
[20] S.P. Sethi, Q. Zhang, Hierarchical Decision Making in Stochastic Manufacturing Systems, Birkhäuser, Boston, 1994.
[21] P. Shi, M. Fu, C.E. de Souza, Loop transfer recovery for systems under sampled measurements, IEE Proc. Control Theory Appl. 143 (1996) 333–337.
[22] P. Shi, E.K. Boukas, H_∞ control for Markovian jumping linear systems with parametric uncertainty, J. Optim. Theory Appl. 95 (1997) 75–99.
[23] P. Shi, C.E. de Souza, L. Xie, Bounded real lemma for linear systems with finite discrete jumps, Internat. J. Control 66 (1997) 145–159.
[24] P. Shi, Filtering on sampled-data systems with parametric uncertainty, IEEE Trans. Automat. Control 43 (1998) 1022–1027.
[25] P. Shi, S.P. Shue, Robust H_∞ control for linear discrete-time systems with norm-bounded nonlinear uncertainties, IEEE Trans. Automat. Control 44 (1999) 108–111.
[26] C.E. de Souza, M.D. Fragoso, H_∞ control for linear systems with Markovian jumping parameters, Control Theory Adv. Technol. 9 (1993) 457–466.
[27] J.C. Willems, Dissipative dynamical systems, part I: general theory, Arch. Rational Mech. Anal. 45 (1972) 321–351.
[28] L. Xie, P. Shi, C.E. de Souza, On designing controllers for a class of uncertain sampled-data nonlinear systems, IEE Proc. Control Theory Appl. 140 (1993) 119–126.